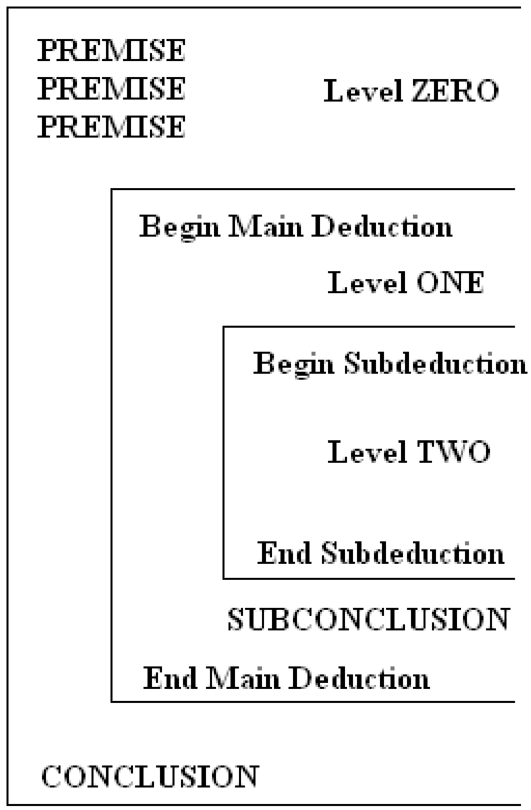
CHAPTER NINE

Introducing Subdeductions

9.1 Deduction Levels

Advanced deductions use a structural tool called the *deduction level.* The deductions you have carried out so far have all taken place at a single level; in this chapter you will learn how to introduce multiple levels into your deductions and use these deeper levels to unlock logical problems that would be extremely difficult, if not impossible, to solve when limited to a single level. The premises of an argument, when placed on the deduction sheet, are at *level zero.* The DEDUCE line that comes right after the premises introduces *level one,* which is the level of the *main deduction.* Subdeductions are nested within level one at levels two and higher, as the schematic diagram below illustrates.



In this chapter we focus on developing a deeper understanding of more complex logical deductions. You may never expect to busy yourself creating MEFs for complicated English arguments, but there are three valuable by-products of learning the methods of advanced deduction.

1. Certain strategies available only by understanding the idea of subdeductions are related to very important techniques of argument and persuasion in "informal logic," the topic of the next chapter.
2. There are advanced fields of study, including philosophy, computer programming and mathematics, where an understanding of these methods is significant.
3. The concepts of using structural modules and sub-modules, as exemplified here, carry over into the general area of organizing for problem solving in any field.

9.1.1 Graphic Representation of a Deduction

In order to carry out a deduction that has more than one level, there has to be a way of distinguishing graphically between the levels. When a subdeduction is complete, there also has to be a graphic method of indicating that it is completed and that its lines are no longer usable. Take a moment to compare the deduction shown below with the diagram on the previous page.

1. PR		A or (B and nC)
2. PR		(A or B) then (D or nC)
DD	X	C then D
3. SN, NC	X	C and nD
4. CJ 3	X	C
5. CJ 3	X	nD
DD	XX	A or B
6. SN,DM	XX	nA and nB
7. CJ 6	XX	nA
8. AD 1,7	XX	(B and nC)
9. CJ 8	XX	nC
10. SC 4!9	X	A or B
11. SE 2,10	X	D or nC
12. AD 5,11	X	nC
13. CC 12!4		C then D

As with the differing mathematical symbols used in traditional logic texts, different texts also represent levels of a deduction using different graphic methods. Our method is similar to some of these, but we have found that the graphics of our method are very intuitive and. The procedure is simple and easy to understand. After comparing the example on the left with the diagram on the previous page, which lines are at level zero, which at level one, and which at level two?

THE INDEX. The level of any line is indicated by the *index.* To create the index, you write an *index mark* after the justification to indicate the level at which the line is written. An index mark is an X-mark, one X for each level. The premises and conclusion of an argument are always at level zero, so they have a *null index,* that is, no X-mark at all. The main deduction is at level one, so every line of the main deduction has a single X-mark. The first subdeduction will be at level two, so a line in level two will have two X-marks in a row.

SCORING. As you can see in the example, all the index marks of the completed deduction have lines drawn through them. These indexes have been "scored." Index marks may be *scored* or *unscored,* depending upon the status of the line. When the lines of the deduction are usable, the index is unscored. A scored index is an index that has a line drawn through the center of at least one of the X's in the index, either horizontally or vertically. Any line that has a scored index is called a "dead'" line; it is no longer in use in the deduction. Only lines with unscored indexes may be used in the deduction. In the example, since the deduction is complete, the only remaining usable lines are the ones at level zero, which are lines 1,2 and 13 (the premises and the conclusion). In what follows we will explain when index marks are written, how they are written, and how they are scored.

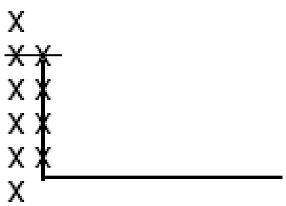
INCREASING THE LEVEL: THE DEDUCE LINE. The only way to increase the level in a deduction is to write a DEDUCE line. As you can see from the example, the main deduction begins right after the premises, that is, it begins with your first DEDUCE line, which raises the level from level zero to level one. As always, your first DEDUCE line states the desired conclusion of the argument. Since it introduces level one, the first DEDUCE line will have a level one index, that is, a single X-mark. Also, the index for any DEDUCE line is *always scored horizontally as soon as it is written,* as an indication that the DEDUCE line is unusable. When the level one DEDUCE line n the example above was written, it would have looked like this.

DD X (C then D)

In advanced deductions you may write more than one DEDUCE line. This is the way you introduce subdeductions into the deduction. Each time you write a new DEDUCE line after the first one, you increase the level by one, which means you add one X-mark to the current index. DEDUCE lines for subdeductions introduce the desired conclusion of that particular subdeduction. Like the first DEDUCE line, the index is also pre-scored (unusable). So when the level two line in the example was written, it would have looked like this.

DD XX (A or B)

DECREASING THE LEVEL: THE CONCLUDE AND SUBCLUDE LINES. The only way to decrease the level in a subdeduction is to end the subdeduction. When you end the main deduction, you also decrease the level (to level zero). The main deduction comes to an end when you write the CONCLUDE line (CC). The index for the CONCLUDE line will be one less than the level of the main deduction, that is, at level zero (see the diagram above). Subdeductions come to an end by writing a SUBCLUDE (subconclusion) line (SC). The index for any SUBCLUDE line will always be one less than the index for the level you are ending. For example, if you are ending a subdeduction at level three, the index for the SUBCLUDE line will be two X-marks. (You will see examples in a moment.)



VERTICAL SCORING. Whenever a particular subdeduction comes to an end, all the lines of that subdeduction become unusable, and must be scored by drawing a *vertical* line through the rightmost X-marks of all the indexes in the lines of the subdeduction, and then out horizontally just above the SUBCLUDE line to enclose all the lines of the subdeduction in an L-shaped bracket (see graphic at left). Because all the indexes for all the lines in the subdeduction will now be scored, the entire subdeduction is rendered unusable. However, just below the horizontal line will be the subconclusion at one less level, and it is now usable. You have traded the lines of the subdeduction for the SUBCLUDE line. (In the example above, you have traded lines 6 to 9 for line 10.) For indexes above level one, until the main deduction is complete, this will leave some of the X's in the columns with no lines through them (see the diagram on the left), but the lines that are vertically scored are "dead" nevertheless. It only takes a single scored X in an index to render the line unusable. You will see many examples of both horizontal and vertical scoring in what follows.

9.2 Deducing Tautology Patterns

In Section 7.4 you were introduced to a logical principle called CONLIM (the contradiction limit). CONLIM is just one of many *tautology patterns* that exist. Tautology patterns are MEF patterns that are always true. Because a tautology pattern is always true, any MEF that is an instance of a tautology pattern may be introduced directly into a deduction at any time. Some tautology patterns are of direct practical use in deductions, while others are interesting and informative in a variety of ways.

Tautology patterns can be proven true by an analysis of their truth conditions, but this is sometimes very tedious and uninformative (especially when the tautology is fairly complicated). In this text you will learn how to validate tautology patterns through deduction. This works because the truth conditions are already built into our definitions of the connectors. Deducing tautology patterns can be an interesting and helpful practice, because tautologies express different facets of the logical relationships that lie at the heart of deductive linkage. A deduction for a tautology pattern often reveals rhythmic characteristics that amount to a kind of "unfolding" of logical patterns and can lead to valuable logical insight.

Deductions for tautology patterns have *no premises,* because a tautology is true regardless of the truth or falsity of other statements. A tautology deduction begins with the DEDUCE line. All the information used to deduce the tautology pattern must be drawn from the SUPNOT line or from other tautologies. The ability to introduce new levels into a deduction is especially helpful when deducing tautology patterns. To make things a little easier, however, the first tautology patterns we will deduce as examples will be very simple ones that do not require subdeductions. We select for our examples in this section four tautology patterns; two of them express what can happen when a given statement is true, and two express what can happen when a statement is false. The first two are very useful in more advanced deductions. They are called ADDALT (AA) and ADDSUF (AS) respectively. We start with ADDALT (Add an Alternate). ADDALT conveys the idea that whenever a statement is *true,* any alternation having that statement as one of its alternates must also be true (regardless of what the other alternate is). ADDALT says that you can ``add" an alternate to any true statement. Below is the start of a deduction of ADDALT.

	DD	~~X~~	A then (A or B)
1.	SN	X	n[A then (A or B)
2.	NC 1	X	A and n(A or B)
3.	CJ 2	X	A
4.	CJ 2	X	n(A or B)

Study the lines of the deduction very carefully. Note how the DEDUCE line index is pre-scored, and how the indexes for the other lines stay at the same level as that of the DEDUCE line, but are not yet scored. In lines 2 to 4 we have followed the proper deduction rhythm for deducing a conditional by using SUPNOT, which is NC CJ CJ (see section 7.7). Now that the rhythm has played itself out, we must do something with line 4. The obvious (and standard) step is to use DEMORG.

The DEMORG step in line 5 allows us to use CJ again. A single application is all that is needed to arrive at a definitive solution. Do you see it? Line 6 directly contradicts line 3! We conclude by indirect deduction. After writing the CONCLUDE line, the indexes are scored as described earlier. A vertical line scores down from the DD line to the final index for the level, and then a horizontal line extends out beneath the last MEF before the CC line. (Isn't that cool?)

	DD	A then (A or B)
1.	SN	n[A then (A or B)
2.	NC 1	A and n(A or B)
3.	CJ 2	A
4.	CJ 2	n(A or B)
5.	DM 4	nA and nB
6.	CJ 5	nA
7.	CC 3!6	A then (A or B)

The second tautology pattern, ADDSUF, expresses the idea that when a statement is *true,* any conditional having that statement as its necessary condition is true regardless of what the sufficient condition is. Thus the effect of ADDSUF is that you may "add" a sufficient condition to any true statement. ADDSUF is a nested conditional, so the full rhythm NC CJ CJ, NC CJ CJ, applies to it. however, we do not need to complete the rhythm with the final CJ step, because a contradiction already appears (lines 3 and 6 below). We show the deduction below, completed and properly scored. Notice the scoring. When was the DD index scored, and when were the remaining indexes scored vertically? Notice that we have "traded" lines 1 through 6 for line 7. All the previous lines are now "dead" because their indexes are scored.

	DD	A then (B then A)
1.	SN	n[A then (B then A)]
2.	NC	A and n(B then A)
3.	CJ 2	A
4.	CJ 2	n(B then A)
5.	NC 4	B and nA
6.	CJ 5	nA
7.	CC 3!6	A then (B then A)

ADDSUF AND CAUSALITY. We pause here for a moment to recall our discussion in Chapter 4 about conditionals and statements of cause. We said there that although statements of cause, such as "If that match is lit, there will be an explosion," may be reasonably represented by our "then" connector, there are many conditionals that do not represent causal relations at all, but which have a different logical role to play in our deductions. If, within the confines of a deduction, we use ADDSUF to produce a conditional that makes no sense as a statement of cause, but whose necessary condition is true, we are not making a peculiar or outlandish *causal* claim.

For example, suppose within a deduction you find that a sentence "Joan is a veterinarian" is true. Then by ADDSUF you could theoretically state "If Lassie loves this bone then Joan is a veterinarian." As a statement of cause it is nonsense: What bones Lassie loves, or hates, have nothing to do with Joan being a veterinarian. But the conditional is not making a statement of cause. All the conditional is really saying is that Joan's being a veterinarian is true, and it is true regardless of what else is true. You might wonder, then, what would be the point of using ADDSUF! But it turns out that sometimes (rarely) within a deduction, the ADDSUF move can be helpful. If you are interested in this point, take another look at the discussion of cause in Chapter Four.

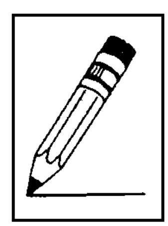
ANDLIM (the AND limit) expresses the idea that when a statement is false, any conjunction containing it must also be false. SUFLIM (the sufficient condition limit) expresses the idea that a conditional is true if its sufficient condition is false, regardless of what the necessary condition may be (see section 4.3.1). In both of these deductions we used the rhythm NC CJ CJ. The first one, below, is ANDLIM.

```
    DD      X    nA then n(A and B)
1.  SN      X    n[ nA then n(A and B) ]
2.  NC 1    X    nA and (A and B)
3.  CJ 2    X    nA
4.  CJ 2    X     A
            |__________________
5.  CC 3!4       nA then n(A and B)
```

SUFLIM is a nested conditional, so we use the double rhythm NC CJ CJ NC CJ CJ, but we found that the last CJ step of the rhythm was not needed.

```
    DD      X    nA then (A then B)
1.  SN      X    n[ nA then (A then B)]
2.  NC 1    X    nA and n(A then B)
3.  CJ 2    X    nA
4.  CJ 2    X    n(A then B)
5.  NC 4    X    A and nB
6.  CJ 5    X    A
            |__________________
7.  CC 3!6       nA then (A then B)
```

The ease with which we found contradictions in these deductions reveals the secret of any tautology deduction: The SUPNOT line for a tautology must *always* contain a contradiction, because a tautology is always true. All that is needed is a way to bring the contradiction out into the open. Sometimes the contradiction is very easy to find, but for more complex problems it takes considerable ingenuity. The deductions for many useful and interesting tautologies may require you to write creative DEDUCE lines and produce tricky logical rhythms. In the next chapter we will introduce you to some very powerful tautology patterns called *biconditionals,* and show in detail how levels above the main deduction can be used to prove that they are true.



STUDY PROBLEMS. Below you will find a group of easy tautology patterns you can use to get some practice. None of them require going beyond level one. Create deductions for these tautology patterns using index marks and scoring lines as shown in the above examples. Some of the tautologies have names, which are shown on the right. When deducing a tautology pattern, do not use that same tautology to prove itself! That would make things too easy! For example, don't use TRANSPOSE to prove problem 9, since the tautology pattern of problem 9 is TRANSPOSE. Also, remember to use the double rhythm NC, CJ, CJ, NC, CJ, CJ for nested conditionals. The names of most of these tautologies are shown on the right.

1.	*(A then A)*	REPEAT (R)
2.	*n(A and nA)*	NONCON (NN) (Non-contradiction)
3.	*(A or nA)*	EXMID (EM) (Excluded Middle)
4.	*A then (A and A)*	REDUND (RD) (Redundancy)
5.	*A then (A or A)*	REDUND (RD)
6.	*(A and A) then A*	REDUND (RD)
7.	*(A or A) then A*	REDUND (RD)
8.	*[A then (A then B)] then (A then B)*	

9.	*(A then nB) then (B then nA)*	TRANS (TR) (Transpose)
10.	*A then (nA then B)*	
11.	*(nA then A) then A*	
12.	*(A then nA) then nA*	
13.	*(A then B) then [(B then C) then (A then C)]*	SYL (Syllogism)
14.	*(B then C) then [(A then B) then (A then C)]*	SYL
15.	*[A then (B then C)] then [(A then B) then (A then C)]*	DISTR (DR) (Distribute)
16.	*(A and B) then [(A or C) and (B or C)]*	DISTR (DR) (Distribute)
17.	*[A then (B then C)] then [B then (A then C)]*	
18.	*(A and B) then (B and A)*	COMMUT (CM)
19.	*[A and (A then B)] then B*	
20.	*[nA and (A or B)] then B*	
21.	*[nB and (A then B)] then nA*	
22.	*[(A and B) then C] then [(A and nC) then nB]*	

Answers to the odd numbered study problems are in the appendix.

9.3 Using Tautologies in Deductions

Why are the tautology patterns in the study problems above written in italic type? Because they do not represent particular statements, but are rules that can represent an indefinite number of statements. Instances of tautology patterns are, in effect, "free premises." They are always available for use in deductions. Instead of listing tautologies as premises, however, you may simply write the tautology in as a line of your deduction when you need it. The justification for the line will be the name of the tautology (or its two-letter abbreviation). If the tautology does not have a name, or you do not remember it, write TAUT or TT for ``Tautology."

For example, a clever way to deduce CONLIM uses an instance of the tautology pattern Non-Contradiction or NONCON (NN), (study problem 2 above), and an instance of the tautology pattern ADDSUF. Study lines 1 and 2 below carefully.

DD	X	(A and nA) then B
1. NONCON	X	n(A and nA)
2. ADDSUF	X	nB then n(A and nA)
3. TRANS 2	X	(A and nA) then B
4. CC 3		(A and nA) then B

This is a rather clever deduction because the way to go about it might not occur to anyone without some experience. But once the main tautologies become familiar it is really not so amazing. Line 1 above enters the tautology of Non-contradiction (NONCON). Line 2 uses ADDSUF to add a sufficient condition to line 1. Applying transposition in line 3 completes the deduction. Although this deduction is very short, you can create an even shorter deduction of CONLIM (and a more informative one) by using SUPNOT. Try this on your own to see how the SUPNOT line for a tautology deduction necessarily contains a contradiction.

CONDITIONAL TAUTOLOGIES. Many tautologies are conditionals. Conditional tautologies like ANDLIM are useful in combination with a SUFEST step. For example, study the sequence of lines below.

7.	**CONJUNCT 3**	**X**	**nA**
8.	**ANDLIM**	**X**	**nA then n(A and C)**
9.	**SUFEST 8,7**	**X**	**n(A and C)**

After establishing nA in line 7, an instance of ANDLIM is introduced in line 8, using C as the second conjunct (any conjunction containing A is false, regardless of what the other conjunct is). No line reference is needed, because the ANDLIM tautology is not being derived from any previous lines. Because ANDLIM is a conditional, lines 7 and 8 together create a SUFEST step with the result shown in line 9. These moves would be useful if the result on line 9 had some further role to play in the deduction. It takes knowledge of the overall demands of the deduction to know just what instance of the tautology to create.

As a shortcut, you are permitted to skip the line that adds the entire tautology and simply go right to the result. This way you can apply a conditional tautology without actually entering the entire tautology. Instead of writing a line with the entire tautology on it and then writing the SUFEST (SE) line afterward, you may simply enter the result of applying SUFEST to the conditional tautology, as shown in line 8 below.

7.	**CONJUNCT 3**	**X**	**nA**
8.	**ANDLIM 7**	**X**	**n(A and C)**

When you use this shortcut the justification must have a line reference after it, to indicate that you are using the statement on that line together with the tautology to produce the desired result.

9.4 Subdeductions

Now at last we come to subdeductions! Advanced deductions involve subdeductions, which are introduced by adding one or more DEDUCE lines after the main deduction has begun. This of course introduces a change of level. (The larger picture in terms of general problem-solving is that when the main problem to be solved is proving difficult, sometimes identifying and solving a subproblem functionally related to the main problem will move the process forward. For example, in working on developing fuel cells for automobiles powered by hydrogen, finding efficient ways to produce hydrogen from inexpensive materials is a subproblem that will support the larger effort.)

> *A DEDUCE line will have as its line statement some MEF that you believe will be useful in furthering the main deduction.*

Subdeductions are often used when dealing with especially difficult arguments that do not have obvious solution routes. A DEDUCE line that introduces a subdeduction will have as its line statement some MEF that you believe will be useful in furthering the main deduction, but that is not available by any obvious series of steps. The new DEDUCE line does not establish the line statement, but merely states a *new* goal that is to be established by completing the subdeduction. Here is an example.

9.	**SE 1,4**	**X**	**(nA or C) and nD**
10.	**CJ 9**	**X**	**nA or C**

What can we do now? Well, line 10 is an alternation. The way you can get something from an alternation is to have the denial of one of the alternates, and then use ALTDEN. So a possibility is to try to show that one of the alternates of line 10 is false:

9.	**SE 1,4**	**X**	**(nA or C) and nD**
10.	**CJ 9**	**X**	**nA or C**
	DD	**~~XX~~**	**nC**

Now the subconclusion nC, if we can get it, would be useful because it would work with line 10 to produce an ALTDEN step. But will you be able to complete this subdeduction? There is *no restriction* as to when you may write a DEDUCE line, how many you may write, or what line statement you may place on a DEDUCE line. This flexibility makes adding DEDUCE lines for subdeductions a potentially creative process. However, there are some general strategies for writing subdeduction DEDUCE lines that serve as reliable guidelines in most situations. We will give these strategy hints here, and in the next sections we will discuss their application to deductions.

STRATEGY HINTS. When your main deduction begins with a SUPNOT strategy, higher level DEDUCE lines work especially well. This is because your earlier SUPNOT line, when you are trying to deduce a tautology, introduces contradictory material that will make the subdeduction easier to carry out. It is a good idea, however, to wait until you have done all you can with your premises and your initial SUPNOT line (including separating all CONJUNCTS and removing any compound negations by using DEMORG or NEGCOND) before you introduce a new DEDUCE line.

> *Wait until you have done all you can with your premises and your initial SUPNOT line before you introduce a new DEDUCE line.*

After you have ``unpacked" your initial SUPNOT line by following the appropriate rhythm, if you cannot see what to do next, try writing a DEDUCE line for some statement that will *combine with some other usable line* in the deduction to produce a useful result. We list the three most common possibilities below as strategy hints for subdeductions.

1. **TO USE A CONDITIONAL:** Write a DEDUCE line for the sufficient condition of the conditional. Suppose you have [(A or B) then nC] on a usable line. To use the conditional you need its sufficient condition (A or B). So write a DEDUCE line for (A or B). When you get it, use it for a SUFEST step with your conditional.

2. **TO USE AN ALTERNATION:** Write a DEDUCE line that denies one of the alternates. For example, you have (nA or C) on a usable line. To use the alternation you need the denial of one of its alternates. Write a DEDUCE line for A, or a DEDUCE line for nC. When you get either of these, use it for an ALTDEN step with your alternation.

3. **TO FORCE A CONTRADICTION:** When you are working beneath a SUPNOT line, write a DEDUCE line for a MEF that will contradict some other unscored MEF in the deduction. For example, you have nA on a usable line. Write a DEDUCE line for A. If you can complete the subdeduction, you will have "forced out" a contradiction and will be able to end your original level.

WHEN TO WRITE A DEDUCE LINE: Write a DEDUCE line for a subdeduction only when you have a clear idea of why you need the statement you are requesting, and what you will do with it after you get it. Follow the guidelines above to determine what kind of DEDUCE line you need.

WHEN *NOT* TO WRITE A DEDUCE LINE: Never write a DEDUCE line for a statement that is available elsewhere, such as a premise (surprisingly, this is a frequent error on the part of beginners). Also, never write a DEDUCE line for a statement you have "made up" because what you really want is the SUPNOT line that comes after it. For example, if you need nA to finish your main deduction, do not arbitrarily write a DEDUCE line for A, thinking that the SUPNOT line next will give you nA. It will, but the nA you get will be dead before you know it. The only thing in the subdeduction you should want to obtain is the DEDUCE line statement. All of the remaining statements in the subdeduction will be dead after you end the level, so they will never be of use in carrying out the main deduction.

The only thing in the subdeduction you should want to obtain is the DEDUCE line statement.

9.5 Subdeductions in Arguments

Got all that? Well then, now we are ready to look at some actual arguments, armed with these new skills. Subdeductions are used in arguments as well as for deducing tautologies, but since arguments have premises, you have more to work with than a single SUPNOT line. When deducing an argument, you should be aware of the nature of the premises and be ready to write DEDUCE lines that will help you use the premises in case you cannot find an obvious way to use them. The *strategy hints* in Chapter Seven will serve as your guide in starting the deduction, and those on this page will help you to create the proper subdeductions as needed. Following Chapter Seven, we start with SN NC CJ CJ rhythm. Suppose you have begun the following argument: Either this sauce contains fresh tomatoes, or the chef is confused and is not from Italy. If either this sauce contains fresh tomatoes or the chef is confused, then either he buys fresh produce or he is certainly no Italian! Therefore, the chef is from Italy and he buys fresh produce.

1.	**PR**		**A or (B and nC)**
2.	**PR**		**(A or B) then (D or nC)**
	DD	**~~X~~**	**C then D.**
3.	**SN,NC**	**X**	**C and nD**
4.	**CJ 3**	**X**	**C**
5.	**CJ 3**	**X**	**nD**

If you had to remain on level one, this might seem rather difficult, but it is easily solved by following suggestion 1 above and introducing a subdeduction. You have a conditional premise on line 2, so try to DEDUCE the sufficient condition of that premise. Watch how easily the subdeduction is completed.

1.	PR		A or (B and nC)
2.	PR		(A or B) then (D or nC)
	DD	~~X~~	C then D
3.	SN, NC	X	C and nD
4.	CJ 3	X	C
5.	CJ 3	X	nD
	DD	~~XX~~	A or B
6.	SN,DM	XX	nA and nB
7.	CJ 6	XX	nA
8.	AD 1,7	XX	(B and nC)
9.	CJ 8	XX	nC
10.	SC 4!9	X	A or B

Study the subdeduction above very carefully, paying special attention to the following:

1. The vertical scoring for the subdeduction affects only the *rightmost* column of X-marks. Nevertheless lines 6 to 9 are now unusable because if any X in an index is scored the line becomes unuseable. But lines 1,2,3,4,5 and 10 are usable because their indexes are unscored..

2. The SUBCLUDE (SC) line reduces the index from level 2 back to level 1 (line 10).

3. The SUBCLUDE (SC) line adds as a line statement the desired DEDUCE line for the subdeduction (A or B). But now it is at level one instead of trapped *inside* the subdeduction. (We chose to deduce this line because it establishes the sufficient condition of premise 2.)

Sometimes beginners are hesitant to start subdeductions because they feel that a deduction becomes more complicated when it contains more levels. But actually there is a trade-off: Although the overall strategy plan of the deduction becomes more elaborate, each individual subdeduction within that strategy plan is usually very easy to complete. Thus in the long run subdeductions make the overall deduction much easier.

The important thing in using subdeductions is to understand the basic requirements of the problem and to choose additional DEDUCE lines for good reasons. The key to this understanding is contained in the strategy hints in Chapter Seven and in this chapter. Whenever you come to a sticking-point in a deduction, refer back to these hints and see if you can apply them to your current situation. (We have included the Chapter 7 hints at the end of this chapter.)

Returning to the example, the level one index on line 10 is unscored, so you may now *use* line 10 in the subdeduction. This usable line is what you get in exchange for lines 6-9, which are now "dead." The value of this method of scoring a subdeduction is that any individual unscored indexes (no scoring on any X-marks) tell you exactly what lines are still usable. This gives you an ongoing visual check on the progress and organization of your deduction.

With your SUBCLUDE in line 10 you have accomplished your sub-goal, which was to get the sufficient condition of the conditional waiting to be used in line 2. The expected SUFEST step using line 2 follows in line 11. After that, ending the deduction is easy. The completed deduction is on the next page. After the final scoring, both columns of X-marks are vertically scored. The only usable lines now are the premises and the conclusion (i.e., the argument).

1.	PR		A or (B and nC)
2.	PR		(A or B) then (D or nC)
	DD	X	C then D
3.	SN, NC	X	C and nD
4.	CJ 3	X	C
5.	CJ 3	X	nD
	DD	XX	A or B
6.	SN,DM	XX	nA and nB
7.	CJ 6	XX	nA
8.	AD 1,7	XX	(B and nC)
9.	CJ 8	XX	nC
10.	SC 4!9	X	A or B
11.	SE 2,10	X	D or nC
12.	AD 5,11	X	nC
13.	CC 12!4		C then D

9.6 Conditional Deduction

In Chapter Seven we introduced two strategies for deductions: Direct Deduction and Indirect Deduction.. There is another strategy, one which is intuitively satisfying and is used very frequently in all sorts of argumentation as well as in the sciences. This method is called Conditional Deduction, or sometimes “Hypothetical Deduction.” It is so important in the sciences that it forms the basis for experimental predictions intended to test a theory.

When the DD line is a conditional, you may write a SUPPOSE SUFFICIENT (SUPSUF) (SS) line. Like the supposition for an indirect deduction, the supposition line must be written directly beneath the DEDUCE line, where the DEDUCE line is a conditional and the supposition is the sufficient condition of that conditional, as in the example below.

DD	**X**	**(nA then A) then A**
1. SUPSUF	**X**	**(nA then A)**

The purpose of the SUPSUF line is to eventually deduce the necessary condition of the DD line after supposing that the sufficient is true.. If you can come up with the necessary condition, the deduction is complete. In a tautology like the one above this happens very quickly if you immediately write a level 2 DD line, like this:

	DD	X	(nA then A) then A
1.	SUPSUF	X	(nA then A)
	DD	XX	A
2.	SUPNOT	XX	nA
3.	SE 1,2	XX	A
4.	SC 2!3	X	A
5.	CC 4		(nA then A) then A

Works like magic! The notation for the conclusion line is CC followed by the number of the line establishing the *necessary condition* of the original DEDUCE line. That line must be at the level of the original DEDUCE line. The ‘A’ in line 3 above only closes level 2. It is line 4 that allows closure of level 1 for a conditional deduction. What has happened here is that upon supposing the sufficient of the conditional is true, you were able to deduce the necessary *at the same level*. Since that is what the conditional says, you have established the conditional itself. In argument, someone might want to prove that “If John decides to go to Africa, he will have a difficult time,” the procedure would be to assume, for the sake of the argument, that John does decide to go to Africa, and then (based on other established considerations) show that this would land John in various difficulties. It is, in essence, a strategic move that is sometimes called a “what if” strategy. This sort of reasoning appears very often in informal argumentation, and is also a move that can be used incorrectly – particularly if the fallacies of supposing the sufficient is true, or supposing the necessary is false, are involved, as we shall see in the next chapter.

> STRATEGY SUGGESTION: When a DEDUCE line is a conditional, try the SUPSUF strategy unless another strategy such as SUPNOT looks obvious to succeed.

9.7 Deduction by Separation of Cases

There is yet one more important strategy moves to be explained. We have come to a point where we may introduce one of the most powerful argument strategies used in either formal or informal argumentation. This strategy is called "separation of cases," or sometimes "constructive dilemma." If, in a deduction, you have an alternation such as (A or B), and if you can show that *either* one of the alternates implies some further statement that you require to complete the deduction, you can use the rule of separation of cases to get the desired result. The justification for this rule is the following tautology, which we will add to our list of 22 tautologies given in section 9.2.

23. *[(A or B) and (A then C) and (B then C)] then C* CASES (CS)

As this may look a bit obscure or complicated at first glance, and since it is a tautology, let's re-state it as an argument.

1.	**PR**		**(A or B)**
2.	**PR**		**(A then C)**
3.	**PR**		**(B then C)**
	DD	**X**	**C**

Can you see how this would go? What strategy would you suggest? Well, you can't use conditional deduction because the DD line is not a conditional. And you can't use direct deduction because there are no direct logical connections between the premises. So the only choice would seem to be the SUPNOT strategy. And watch how fast it goes!

1.	PR		(A or B)
2.	PR		(A then C)
3.	PR		(B then C)
	DD	X	C
4.	SN	X	nC
5.	NC 3,4	X	nB
6.	AD 1,5	X	A
7.	SE 2,6	X	C
8.	CC 4!7		C

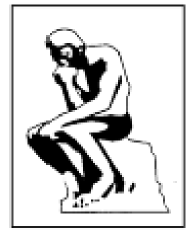
The way the strategy of separation of cases works is to locate, or if necessary establish, some alternation within an argument, and then show that either of the alternatives will lead to the desired conclusion. The two "cases" are two conditionals which each have one of the alternates as the sufficient condition, and which each have the desired conclusion as the necessary condition. In the deduction to the left, the two cases have already been inserted as premises (lines 2 and 3) but in an actual argument you will not likely have the cases immediately available. Instead you can attempt to deduce each case. And since each case will be a conditional, the SUPSUF strategy would be used for deducing each case.

USING EXCLUDED MIDDLE. In our list of tautologies we included the one called "Excluded Middle" (EXMID or EM). This tautology has the form of an alternation, and since it is a tautology it may always be introduced into a deduction. The tautology is, of course, *(A or nA)*. Now if you can show that either of these alternatives will lead to your desired conclusion, you will have a deduction of the conclusion by separation of cases.

On the next page is an English language argument that can be deduced by using separation of cases. Try it at home and bring your result to class for discussion. HINT: After the DD line introduce an appropriate instance of EXMID (EM). (We will be discussing both conditional deduction and deduction by separation of cases in the next chapter.)

Aloysius Campbell has been accepted at two prestigious universities, but he has a problem. He will attend one or the other of these universities. But if he attends the one in northern Scotland, the cold weather will aggravate his delicate constitution and he will be anxious to return home. However, if he does not attend the university in northern Scotland, he will attend the one in Cape Town, in which case his severe allergy condition will disrupt his studies and he will be anxious to return home. Therefore, Aloysius will be anxious to return home.

EXERCISE 14. The problems below are arguments. Not all of them will require subdeductions. If you see more than one way to complete a deduction, write out each possible solution. Answers to starred problems are in the appendix. Remember the strategy suggestions (summarized at the end of this chapter).



1. A then (B then C). A then (C then D). Therefore, A then (B then D).
2.* nA then B. A then B. Therefore, B.
3.* B then nC. nA then C. Therefore, nA then nB.
4. (A then B) then C. nC. Therefore, nB.
5.* (C then D) then A. nD then B. Therefore, nA then B.
6. (B then nnD) then (nC then nD). Therefore, D then C.
7. (D then A) then C. nC. (A then B) then (E then C). Therefore, nE.
8.* (A then B) then (E then C). F then nC. n(D then A). Therefore, F then nE.
9. (A or B) then (C or D). (C or D) then (E and F). (F and A). Therefore, E.
10. (A or B) then C. (D or C) then E. (A or D). nD. Therefore, E.
11.* (A then B) then C. D then (nB then E). Therefore, (C or nE) then (D then C).
12.* [A then (nB and D)] and [(nB and D) then A]. A and (nF then nD). Therefore, nB or F.
13. (A and B) then (C or D). Therefore, [(A then C) or (B then D)].
14. (A or B) then (A and B). (A and B) then (A or B). Therefore, (A then B) and (B then A).
15.* (A then B) or (C then D). Therefore, (A then D) or (C then B).
16.* (A or B) then (C or D). (D or C) then (E and F). A. Therefore, E.
17. (A or B) then (C and D). (C or D) then E. nA then B. Therefore, E.
18. (nA and nB) then (C then B). B then A. nA. Therefore, nC.
19. A then B. B or C. (C and nA) then (D and nA). nB. Therefore, D.
20.* (A and B) then C. D then (B and A). Therefore, nD or C.
21. A then B. B then A. Therefore, (A and B) or (nA and nB).

The next five problems are in English. They are fairly complex, and you will want to watch out for chances to commit the fallacy of the misplaced connector! If you find a sentence is ambiguous as to which is the major connector and which the minor connector(s), write the problem out both ways to see which one might be valid. You will need a full memory pad and you may wish to refer to the flowchart in Chapter Five.

22. Either the President will sign the environmental legislation, or the opposition will filibuster and the legislation will never reach his desk. But if either the President signs the legislation or the opposition filibusters, then either a third party will emerge victorious or the bill will never reach the President's desk. We are forced to conclude that if the bill reaches the President's desk, then a third party will emerge victorious.

23. Either the wagon left the trail at the bend, or it did not leave the trail at the bend and the horses were too tired to go further. If the wagon left the trail at the bend, then at least one of the passengers had to get out, and moreover if the horses were too tired to go further, at least one of the passengers still had to get out. We know for certain, then, that at least one of the passengers had to get out.

24. This medicine is prescribed for the purpose, and either the medicine will work or we will know the reason why. Either the doctor's diagnosis is incorrect, or, if the medicine will work then we have nothing to worry about and if we know the reason why we also have nothing to worry about. So we have nothing to worry about.

25. These tomatoes are ripe or the cook will be angry. Furthermore the cook is either from Sicily or Naples. Therefore the tomatoes are ripe and the cook is from Naples, or the tomatoes are ripe and the cook is from Sicily, or the cook will be angry and she is from Sicily, or the cook will be angry and she is from Naples.

26. Either it is not true that Marianne is a Senior who will not graduate, or if Marianne changes her major then she will spend an extra year in college. But If Marianne changes her major and hates it, then she won't spend an extra year in college. Yet it is not true that either she will not change her major or she will not enjoy it. Alas! The end of the semester is coming and there is no time to waste! We must conclude, I am sorry to say, that either Marianne is not a Senior and the semester is coming to an end, or Marianne is not a Senior and there is no time to waste, or Marianne will graduate and the semester is coming to an end, or Marianne will graduate and there is no time to waste. At least that's the way I see it, but I am getting confused!!

STRATEGY SUGGESTIONS IN THIS CHAPTER

1. TO USE A CONDITIONAL: Write a DEDUCE line for the sufficient condition of the conditional. When you get it, use it for a SUFEST step with your conditional.

2. TO USE AN ALTERNATION: Write a DEDUCE line that denies one of the alternates. When you get either of these, use it for an ALTDEN step with your alternation.

3. TO FORCE A CONTRADICTION: When you are working beneath a SUPNOT line, write a DEDUCE line for a MEF that will contradict some other unscored MEF in the deduction. If you can complete the subdeduction, you will have "forced out" a contradiction and will be able to end your original level.

STRATEGY HINTS FROM CHAPTER 7

STRATEGY HINTS FOR DEDUCTIONS	
DEDUCE line	Strategy
Simple Sentence	Direct Deduction or SUPNOT
Conjunction	Obtain each conjunct separately, then combine them with JOINUP.
Alternation	SUPNOT followed by DM, CJ, CJ
Conditional	SUPNOT followed by NC, CJ, CJ
Any negation	SUPNOT

NEW DEDUCTION METHODS IN THIS CHAPTER

SUPPOSE SUFFICIENT (SUPSUF) (SS). When you have a conditional on a DEDUCE line, write a SUPSUF line for the sufficient condition of the conditional, directly beneath the original DEDUCE line. Then try to obtain the necessary condition of the conditional. If you reach the necessary condition you have proven the conditional so you can score the indexes and write the conditional as a CC or SC line depending on what level your original DEDUCE line for the conditional occupied. (See the example on page 102.)

SEPARATION OF CASES (CASES) (CS). Locate, or if necessary establish, some alternation within an argument, and then show that either of the alternatives will lead to the desired conclusion.